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ABSTRACT

One of the most frequently used research methods in education and psychology involves repeated observations on the same individuals. When sample sizes are relatively small and a multivariate analysis lacks power, there are currently two analytical options in testing time effects. One is to assume a time series structure to these observations, and another is to assume a conventional univariate mixed effect model without a time dependent structure. A restrictive assumption of error sphericity is required for both approaches. If sphericity is not satisfied, the test statistic from either approach is approximately distributed as the "F". The theoretical lowerbound for the sample estimate is shown to be larger when the covariance matrix has a particular type of stationary time series structure (i.e., a serial correlation pattern). A Monte Carlo study was conducted, which indicates that postulating a serial correlation pattern may not have practical advantages over not assuming such a correlational pattern, at least for small sample sizes. The likelihood ratio test for serial correlation frequently rejects the true null hypothesis. This tendency is most pronounced when sample sizes are small. When a serial correlation is high, up to 39% of the covariance matrices sampled from a population covariance matrix with a serial correlation pattern show equal or larger correction factors when such a pattern is not assumed. Six data tables are included. (Author/TJH)

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Fitting a Serial Correlation Pattern to Repeated Observations Lacking Sphericity

Paper presented at the annual meeting of the American Educational
Research Association in Boston (April 20, 1990).

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Running Head: A Serial Correlation

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Fitting a Serial Correlation Pattern to Repeated Observations Lacking Sphericity**Abstract**

One of the most frequently used research methods in education and psychology is to make repeated observations on the same individuals. When sample sizes are relatively small, therefore, a multivariate analysis lacks power, there are currently two viable analytical options in testing time effects (Anderson, Jensen, & Schou, 1981). One is to assume a time series structure to these observations (Azzalini, 1984; Hearne, Clark, & Hatch, 1983; Anderson, 1978; Frekeriksen & Rotondo, 1979) and another is to assume a conventional univariate mixed effect model without a time dependent structure (Davidson, 1972; Collier, Baker, Mandeville, & Hays, 1967; Barcikowski, & Robey, 1984; Greenhouse, & Geisser, 1959).

A restrictive assumption of error sphericity is required for both approaches in examining the time effects. If sphericity is not satisfied, the test statistic from either approach is approximately distributed as the F with $\epsilon\nu_1$ and $\epsilon\nu_2$ degrees of freedom, where ϵ is the correction factor (Box, 1954). The theoretical lowerbound for the sample estimate of ϵ is shown to be larger when the covariance matrix has a particular type of a stationary time series structure, i.e., a serial correlation pattern (Wallenstein & Fleiss, 1979).

A Monte Carlo study suggested that postulating a serial correlation pattern may not have practical advantages over not assuming such a correlational pattern, at least for small sample sizes ($n \leq 24$). The likelihood ratio test for serial correlation (Koopmans, 1942) frequently rejects the true null hypothesis. This tendency is most pronounced when sample sizes are small. Furthermore, when a serial correlation is high ($\rho > .50$), up to 39% of the covariance matrices sampled from a population covariance matrix with a serial correlation pattern show equal or larger correction factors when such a pattern is not assumed. It should be cautioned against indiscriminately fitting a serial correlation pattern in hope of obtaining more power.

Key words: Repeated Measures, Longitudinal studies, Serial Correlation, Markov simplex pattern

Introduction

One of the most frequently used research methods in education and psychology is to make repeated observations on the same individuals. Repeated observations may be sought from each subject in several different contexts. A sample of college students may be asked to take two versions of a same test in a counterbalanced sequence in order to create parallel tests. A group of subjects may be measured at intake, followed by an intervention, then monitored repeatedly at some intervals. Patients may undergo several different drug treatments, where each drug treatment is randomly sequenced (i.e., cross-over designs). Cognitive skills of children may be annually monitored to evaluate individual growth over time. Often the studies are referred to as repeated measures designs, when the repeated factor is an experimentally induced variable; and other studies which are observational in nature are referred to as longitudinal studies. In either case, the response measures from the same individuals are correlated across time (or across treatment levels) and they are assumed to be uncorrelated across different subjects.

Assuming the repeated observations are made on one dependent variable, Y , the data can be analyzed by a multivariate or a univariate method. When sample sizes are relatively small, therefore, a multivariate analysis lacks power, there are currently two viable analytical options in testing time effects (Anderson, Jensen, & Schou, 1981). One is to assume a time series structure to these observations (Azzalini, 1984; Hearne, Clark, & Hatch, 1983; Anderson, 1978; Frekeriksen & Rotondo, 1979) and another is to assume a conventional univariate mixed effect model without imposing a time dependent structure to data (Davidson, 1972; Collier, Baker, Mandeville, & Hays, 1967; Barcikowski, & Robey, 1984; Greenhouse, & Geisser, 1959). Basically a univariate mixed model assumes that the correlations between the observations taken at different time points are not a function of the temporal distance between the two time points.

A time series analysis, on the other hand, typically hypothesizes that the correlation between the observations from two time points is a negative function of the temporal distance between these two time points. That is, observations taken from two immediately adjacent

time points are most highly correlated; and as the amount of time between observations increases, the correlation decreases proportionally.

A restrictive assumption of error sphericity is required for both approaches in examining the time effects. The validity conditions required for a univariate mixed effect model have been well documented (Huynh & Feldt, 1970; Rouanet & Lepine, 1970). When a restrictive assumption of error sphericity is not satisfied, the test statistic from either approach is approximately distributed as the F with $\epsilon\nu_1$ and $\epsilon\nu_2$ degrees of freedom, where ϵ is the correction factor (Box, 1954). The correction factor is shown to be bounded between $1/(t-1)$ and 1, where t is the number of time points. Greenhouse and Geisser (1959) suggested estimating ϵ by a sample covariance matrix, $\hat{\Sigma}$, and adjusting the degrees freedom by $\hat{\epsilon}$. Several simulation studies have shown that the adjusted F - tests adequately control the test size (Collier, Baker, Mandeville, & Hayes, 1967; Rogan, Keselman, & Mendoza, 1979).

Serial correlation or simplex pattern

In applying a serial correlation pattern, we have to estimate the sample variance and correlation. A likelihood ratio test then tests the adequacy of the estimated covariance matrix based on these sample estimates. The sample correction factor $\hat{\epsilon}_p$ can be obtained from the estimated covariance matrix, which is derived from the variance estimate and the correlation estimate. Wallenstein and Fleiss (1979) have shown that if the covariance matrix has a serial correlation pattern (or a simplex pattern), the correction factor obtained from such a patterned matrix will have a larger lowerbound than the correction factor obtained without assuming such a pattern. However, there has been no conclusive evidence that fitting a serial correlation pattern would always produce more powerful results, that is, there will be larger sample correction factors if we fit the simplex pattern to the repeated observations.

Hearne, Clark, and Hatch (1983) have illustrated the LR test for serial correlation (Koopmans, 1942) and computed the correction factor, $\hat{\epsilon}_p$, for their data by first fitting the serial correlation pattern. The recalculation of their data by this author confirmed $\hat{\epsilon}_p = 0.56$ but contrary to the expectation, the correction factor without fitting such a pattern resulted in a

larger correction factor, $\hat{\epsilon} = 0.64$, indicating that the latter approach would have produced a more powerful result than the former approach.

If a certain structure can be postulated for the covariance matrix, a potentially more powerful analysis can be adopted. If, however, such an assumption is erroneous or is erroneously rejected when it is in fact true, the analysis method chosen assuming such a structured covariance matrix may be invalid or may lack power. In addition, the effects of erroneously postulating a serial correlation pattern are not well understood. In educational and psychological research, it is possible to have a patterned correlation matrix somewhat similar to, $\mathbf{V}$ in the AR(1) --Model 1, but the variances may be nonstationary. For example, in studies where learning or maturation effects are involved, the variance may increase if the individual differences at the onset widen with time, i.e., $\sigma_j^2 < \sigma_{j'}^2$ for $j < j'$. In other occasions when the number of errors is the dependent variable in a learning experiment, the variance may actually decrease with time, i.e., $\sigma_j^2 > \sigma_{j'}^2$ for $j > j'$.

Assuming that series of observations on a variable from N subjects are made at t equally spaced time points, the Y_{ij} , the response of subject i at time j can be expressed as $Y_{ij} = \mu + \alpha_i + \beta_j + u_{ij}$, where μ is the overall mean, α_i is the subject effect, β_j is the occasion effect, and u_{ij} is the error. If we assume that Y_{ij} to follow a particular type of a stationary first-order autoregressive process, AR(1), the covariance matrix will have a simplex pattern.

Covariance matrices with a serial correlation pattern (or a simplex pattern) imply that the observations have a stationary variance and the correlation between the observations taken at two time points is a negative function of the temporal distance between the two time points. Specifically, the covariance matrix (Σ) with a serial correlation pattern, $\mathbf{R}$, can be expressed as a special case of the following form:

$\Sigma = \sigma^2 (1/1 - \rho^2) \mathbf{V}$, where $\sigma^2 (1/1 - \rho^2)$ is the common variance. In this model, $\mathbf{V}$ is identical to a correlation matrix, $\mathbf{R}$. Here, t responses from each subject form a geometric series. With $|\rho| < 1$, this series converges uniformly.

$$\mathbf{V} = \begin{bmatrix} 1 & \rho & \rho^2 & \dots & \rho^{t-1} \\ \rho & 1 & \rho & \dots & \rho^{t-2} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \rho^{t-1} & \rho^{t-2} & \rho^{t-3} & \dots & 1 \end{bmatrix} \quad \text{Model 1.}$$

$\mathbf{V}$ is of size $t \times t$. The expression is simplified for the covariance matrices with a serial correlation pattern, $\Sigma = \sigma^2 (1/\rho^2) \mathbf{V}$.

It may be argued that the nonstationary model may be more realistic in educational and psychological data. The variances may increase monotonically across time and the increment is a monotonic function of time. The incremental variance first-order autoregressive process, INAR(1), $\Sigma = \sigma_1^2 \phi_j^2 (1/\rho^2) \mathbf{V}$, where ϕ_j^2 is a proportionality constant. For example, by setting $\phi_j^2 = j$, where $j = 1, \dots, t$, we have

$$\mathbf{V} = \begin{bmatrix} \phi_1^2 & \phi_1 \phi_2 \rho & \phi_1 \phi_3 \rho^2 & \dots & \phi_1 \phi_t \rho^{t-1} \\ \phi_1 \phi_2 \rho & \phi_2^2 & \phi_2 \phi_3 \rho & \dots & \phi_2 \phi_t \rho^{t-2} \\ \vdots & \vdots & \phi_3^2 & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \phi_1 \phi_t \rho^{t-1} & \phi_2 \phi_t \rho^{t-2} & \dots & \dots & \phi_t^2 \end{bmatrix} \quad \text{Model 2.}$$

As a third model, we need a nonstationary AR(1) with decreasing variances. Therefore, Model 3 is created which corresponds to Model 2 with a reversed pattern in major diagonal elements. The decremental variance first-order autoregressive process, DEAR(1), can be expressed as $\Sigma = \sigma_1^2 \phi_j^2 (1/\rho^2) \mathbf{V}$, where ϕ_j is a proportionality constant. For example, by specifying $\phi_j^2 = t - j + 1$, where $j = 1, \dots, t$, we have

$$\mathbf{V} = \begin{bmatrix} \phi_1^2 & \phi_1 \phi_2 \rho & \phi_1 \phi_3 \rho^2 & \dots & \phi_1 \phi_t \rho^{t-1} \\ \phi_1 \phi_2 \rho & \phi_2^2 & \phi_2 \phi_3 \rho & \dots & \phi_2 \phi_t \rho^{t-2} \\ \vdots & \vdots & \phi_3^2 & \vdots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \phi_1 \phi_t \rho^{t-1} & \phi_2 \phi_t \rho^{t-2} & \dots & \dots & \phi_t^2 \end{bmatrix} \quad \text{Model 3.}$$

These three models will be examined as plausible time series structures in repeated observations.

A likelihood ratio test for the serial correlation pattern

Based on the work of Koopmans(1942), Hearne, et al. (1983, pp. 240-241) have presented a likelihood ratio test statistic for examining whether a given covariance matrix has a serial correlation pattern. For simplicity, we assume a single group of subjects with repeated observations taken at equal intervals.

Let the likelihood ratio test be expressed as $\lambda = L(\hat{\omega})/L(\hat{\Omega})$, where the numerator likelihood estimates corresponds to the null hypothesis and the denominator corresponds to the general unrestricted space. We express y_{ij} as a response in the form of a deviation from the sample mean. First, we need to solve for $\hat{\sigma}^2$ and $\hat{\rho}$ by two partial differential equations: $(\partial/\partial\rho) \ln L(\omega) = 0$ and $(\partial/\partial\sigma^2) \ln L(\omega) = 0$.

From $(\partial/\partial\rho) \ln L(\omega) = 0$, we have

$$-N\rho\sigma^2 + (1 - \rho^2) (A_1 - \rho A_2) = 0, \quad (1)$$

where

$$A_1 = \frac{N}{N-1} \sum_{i=1}^N \sum_{j=2}^t y_{ij} y_{i(j-1)}, \text{ and} \quad (2)$$

$$A_2 = \frac{N}{N-1} \sum_{i=1}^N \sum_{j=2}^{t-1} y_{ij}^2. \quad (3)$$

From $(\partial/\partial\sigma^2) \ln L(\omega) = 0$, we then have

$$-Nt\sigma^2 + A_2 + A_3 - 2A_1\rho + A_2\rho^2 = 0, \quad (4)$$

where

$$A_3 = \frac{N}{N-1} \sum_{i=1}^N (y_{i1}^2 + y_{it}^2). \quad (5)$$

By solving (4) for σ^2 and substituting the result into (1), we obtain

$$(1 - 1/t) A_2 \rho^3 - (1 - 2/t) A_1 \rho^2 - \{ (1 + 1/t) A_2 + A_3/t \} \rho + A_1 = 0, \quad (6)$$

which is a polynomial function of ρ . By solving this equation, ρ is obtained and this solution in turn is used to solve for σ^2 . The likelihood ratio test statistic, $-2 \ln \ell$, is approximately distributed as χ^2 with $1/2 t(t+1) - 2$ df. Hearne et al.'s A_1 , A_2 , and A_3 correspond to Nm , Nn , and $N\ell$, respectively, in Koopman's expressions.

Purpose of the Study

The purpose of this study is to determine practical advantages of fitting a serial correlation pattern to repeated observations over not assuming such a patterned correlation when the population covariance matrix indeed has a serial correlation pattern. In addition, the affects of erroneously specifying a serial correlation pattern or AR(1) are examined when the variances are nonstationary.

Method

A computer simulation method is used to investigate power advantages of fitting a serial correlation pattern to repeated observations. Random numbers are generated using the GGNSM algorithm in the IMSL (IMSL, 1981). A computer program for this study was first tested for its accuracy using the data and the intermediate computations reported in Hearne et al. (1983). The values of the correction factors obtained by fitting a serial correlation pattern is compared to those obtained without postulating such a pattern. The mean correction factors, from the original covariance matrix and from the estimated simplex pattern, and the rejection rates of the LR test for the simplex pattern are compared. The following conditions are examined: $n = 6$ (for $t=3$ and 5 only), 12, and 24; $\rho = 0.00, 0.25, 0.50, 0.75$, and 0.90 ; and three nominal α levels for the LR test of serial correlation: 0.05, 0.10 and 0.20.

Three types of plausible covariance matrices are investigated. One is a stationary AR(1) with the serial correlation pattern V . The common variance $\sigma^2(1/\{1-\rho^2\})$ was set to $1/\{1-\rho^2\}$. The second covariance pattern had the same correlational pattern but the variances increased over time. The pattern of variance increment was set to 1: 2: 3 for $t = 3$, 1:2:3:4:5 for $t = 5$, and 1:2:3:4:5:6:7 for $t = 7$, simulating a gradual increment. The third covariance pattern had the same correlation pattern, but the variances decreased with time. The pattern of decrement is

obtained by reversing the pattern for the second condition. The covariance patterns from these two types of nonstationary models are slightly different. That is, both models have a gradually decreasing correlation pattern but because the variances are increasing in Model 2, the covariances decrease at a slower pace than in Model 3, where the variances decrease as well as the correlations.

Both approaches in obtaining sample correction factors considered here result in approximate F -distributions with possibly different degrees of freedom. Because the power function for the F considered here is strictly a nondecreasing function of the degrees of freedom, hence, a function of the correction factor alone, it is sufficient to examine the sizes of these correction factors for the empirical power comparison of these two approaches.

Results

The results for data matrices with simplex patterns are summarized in Tables 1 to 3; and the results for the data with an increasing variance pattern are summarized in Tables 4 through 6. The results from the decreasing variance pattern are not shown because they had very similar results as those obtained from the increasing variance pattern. In each of these tables, the sample mean of the correction factors without assuming a simplex pattern, $\hat{\epsilon}$, and the sample mean of the correction factors from the covariance matrices fitted for the simplex pattern, $\hat{\epsilon}_p$, and the theoretical lowerbounds for these corrections factors are reported. The theoretical lowerbound for $\hat{\epsilon}$ is $1/[t-1]$ (Box, 1954); and for $\hat{\epsilon}_p$, $5[t+1]/[2t^2 + 7]$ (Wallenstein & Fleiss, 1979). Wallenstein and Fleiss have also reported the theoretical values for epsilon when the correlation matrix has a simplex pattern (p.231). For $t = 3$, the epsilon values are 1.000, 0.977, 0.925, 0.862, and 0.824 for $\rho = 0.00, 0.25, 0.50, 0.75$, and 0.90 ; for $t = 5$, the values are 1.000, 0.941, 0.809, 0.660, and 0.577; and for $t = 7$, the values are 1.000, 0.923, 0.743, 0.546, and 0.442, respectively. The mean $\hat{\epsilon}_p$ in this study are all very close to those theoretical values (Tables 1, 2, and 3), giving credence to the simulation results.

The results are based on 2000 replications. The proportion in the row $\hat{\epsilon} \geq \hat{\epsilon}_p$ indicates the proportion of replications that showed larger correction factors when the simplex pattern was

not assumed rather than when such a pattern was postulated. A significant Chi-square test indicates that the assumption of a simplex pattern is rejected. Subsequently, a correction factor can be estimated only from a covariance matrix without fitting the serial correlation pattern. The proportions reported are the combined proportions of such rejections and the cases where $\hat{\epsilon} \geq \hat{\epsilon}_p$, excluding these overlapping matrices that are both $\hat{\epsilon} \geq \hat{\epsilon}_p$ and are rejected for the patterned matrix.

For high serial correlations ($\rho > 0.50$), indicating that either the time periods are close together or the behavior or trait that is measured is consistent over time, up to 39% of the covariance matrices sampled from the population covariance matrix with a serial correlation pattern showed equal or larger correction factors when the simplex pattern was not assumed (Table 1). When the time series is short ($t = 3$), the LR test is shown to be extremely sensitive, i.e., it rejects the true null hypothesis of a simplex pattern. When the rejection rates are combined with the cases in which the sample correction factors from the original covariance matrices are larger than the counterparts derived from the postulated simplex structure, they add up to as large as 46%.

Insert Table 1 about here

As the number of time periods increases, the correction factors obtained from the estimated patterned matrices are almost always larger than those obtained from the actual covariance matrices. On the other hand, the empirical rejection rate of the LR test increases dramatically as the number of time points increases. Consequently, researchers will be forced to accept the correction factors obtained from the actual covariance matrices rather than those obtained from the estimated patterned matrices. This implies that the researchers cannot capitalize on the large correction factors from the serially correlated responses when t increases.

One of the disconcerting results is that the LR test is extremely sensitive. It rejects the true null hypothesis of the patterned covariance matrix even when the sample size is increased to $n = 24$. The test size of the LR test when $t = 5$, $n = 24$ and at $\alpha = 0.05$ is 0.097 (Table 2), which is about twice as large as the nominal level of 5%. This over sensitivity even increases as the correlation becomes positive and large. The largest increase, that is, a fourfold increase in rejection rates, can be seen when the results for $\rho = 0.00$ are compared with those for $\rho = 0.90$.

Insert Table 2 about here

When t (the number of time points) increases to 7, the LR test remains still sensitive even with n as large as 24. Even without applying the LR test, as the correlation becomes large and positive ($\rho = 0.90$), the correction factors obtained from the original covariance matrix become larger than the counterparts based on the serial correlation pattern for about 1/3 of the time (Table 3).

Insert Table 3 about here

When the variance is nonstationary, the correction factors from the patterned covariance matrices are pronouncedly larger than the counterparts from the original covariance matrices (Tables 4, 5, and 6). The results from the population covariance matrix with decreasing variances were very similar to those obtained from the covariance pattern with increasing variances. Because these are the occasions in which the simplex pattern does not hold, adopting the correction factors assuming the serial correlation pattern can lead to erroneous results. Furthermore, the power of the LR test is strikingly high. With these two facts in mind, fitting a serial correlation pattern is not advisable when the covariance matrix shows a nonstationary pattern.

Insert Tables 4, 5, and 6 about here

Discussion

Although uncritical extrapolation of the simulation results should be avoided beyond the conditions actually tested, postulating a serial correlation pattern may not have practical advantages over not assuming such a correlational pattern, at least for small sample sizes. First of all, the likelihood ratio test for a serial correlation pattern (Koopmans, 1942) frequently rejects the true null hypothesis. Using liberal type I error levels for this screening test is not advisable because it is already sensitive at $\alpha = 0.05$ level. Secondly, when the serial correlation is high ($\rho > .50$) and the number of time points is small, up to 39% of the covariance matrices sampled from a population covariance matrix with a serial correlation pattern had equal or larger correction factors when such a pattern was not assumed. A practical approach for researchers with repeated observations with small sample sizes and high correlations among the responses may be not to adopt a simplex pattern, even when such a pattern is suspected. Indiscriminately fitting a serial correlation pattern in hope of increasing the power of test should be discouraged.

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Table 1. Mean Sample Correction Factors and the Proportion of Sample Covariance Matrices Producing Larger Correction Factors when the Serial Correlation Pattern is not Assumed: $t=3$

	ρ					Lowerbound correction factor
	0.000	0.250	0.500	0.750	0.900	
<u>$t = 3$ at $n = 6$</u>						
$\hat{\epsilon}$	0.775	0.768	0.755	0.728	0.729	0.500
$\hat{\epsilon}_\rho$	0.961	0.958	0.926	0.874	0.833	0.800
$\hat{\epsilon} \geq \hat{\epsilon}_\rho$	0.077	0.068	0.125	0.163	0.248	
χ^2 sig at .05	0.329	0.310	0.365	0.388	0.464	
χ^2 sig at .10	0.420	0.402	0.465	0.485	0.539	
χ^2 sig at .20	0.554	0.536	0.593	0.607	0.643	
<u>$t = 3$ at $n = 12$</u>						
$\hat{\epsilon}$	0.868	0.857	0.837	0.798	0.782	0.500
$\hat{\epsilon}_\rho$	0.981	0.969	0.928	0.868	0.829	0.800
$\hat{\epsilon} \geq \hat{\epsilon}_\rho$	0.081	0.122	0.213	0.310	0.345	
χ^2 sig at .05	0.202	0.221	0.324	0.415	0.426	
χ^2 sig at .10	0.269	0.293	0.390	0.477	0.472	
χ^2 sig at .20	0.384	0.419	0.507	0.569	0.566	
<u>$t = 3$ at $n = 24$</u>						
$\hat{\epsilon}$	0.928	0.913	0.880	0.836	0.800	0.500
$\hat{\epsilon}_\rho$	0.991	0.973	0.928	0.866	0.826	0.800
$\hat{\epsilon} \geq \hat{\epsilon}_\rho$	0.071	0.175	0.312	0.379	0.386	
χ^2 sig at .05	0.141	0.251	0.375	0.434	0.438	
χ^2 sig at .10	0.199	0.303	0.425	0.480	0.479	
χ^2 sig at .20	0.314	0.415	0.518	0.558	0.551	

Note: The results are based on 2000 replications. Each of the population covariance matrix had a serial correlation pattern. The proportion in the row $\hat{\epsilon} \geq \hat{\epsilon}_\rho$ indicates the proportion of replications that showed larger correction factors when the simplex pattern was not assumed rather than when the pattern was fitted. A significant Chi-square test indicates that the assumption of a simplex pattern was rejected. Subsequently, an epsilon can be estimated only from a covariance matrix without fitting the serial correlation pattern. The proportions reported above are the combined proportion of such rejections and the cases where $\hat{\epsilon} \geq \hat{\epsilon}_\rho$.

Table 2. Mean Sample Correction Factors and the Proportion of Sample Covariance Matrices Producing Larger Correction Factors when the Serial Correlation Pattern is not Assumed: $t=5$

	ρ					Lowerbound correction factor
	0.000	0.250	0.500	0.750	0.900	
<u>$t = 5$ at $n = 6$</u>						
$\hat{\epsilon}$	0.566	0.557	0.532	0.489	0.462	0.250
$\hat{\epsilon}_\rho$	0.953	0.920	0.822	0.628	0.593	0.526
$\hat{\epsilon} \geq \hat{\epsilon}_\rho$	0.001	0.002	0.009	0.054	0.123	
χ^2 sig at .05	0.754	0.734	0.749	0.781	0.806	
χ^2 sig at .10	0.825	0.811	0.818	0.844	0.871	
χ^2 sig at .20	0.898	0.888	0.892	0.900	0.919	
<u>$t = 5$ at $n = 12$</u>						
$\hat{\epsilon}$	0.730	0.708	0.648	0.573	0.523	0.250
$\hat{\epsilon}_\rho$	0.977	0.932	0.813	0.673	0.585	0.526
$\hat{\epsilon} \geq \hat{\epsilon}_\rho$	0.001	0.006	0.047	0.168	0.254	
χ^2 sig at .05	0.220	0.210	0.267	0.367	0.459	
χ^2 sig at .10	0.314	0.321	0.373	0.459	0.567	
χ^2 sig at .20	0.471	0.473	0.511	0.601	0.674	
<u>$t = 5$ at $n = 24$</u>						
$\hat{\epsilon}$	0.846	0.807	0.720	0.618	0.553	0.250
$\hat{\epsilon}_\rho$	0.988	0.937	0.813	0.667	0.580	0.526
$\hat{\epsilon} \geq \hat{\epsilon}_\rho$	0.001	0.013	0.116	0.279	0.348	
χ^2 sig at .05	0.097	0.128	0.222	0.371	0.424	
χ^2 sig at .10	0.165	0.211	0.303	0.426	0.487	
χ^2 sig at .20	0.303	0.339	0.424	0.518	0.571	

Note: The results are based on 2000 replications. Each of the population covariance matrix had a serial correlation pattern. The proportion in the row $\hat{\epsilon} \geq \hat{\epsilon}_\rho$ indicates the proportion of replications that showed larger correction factors when the simplex pattern was not assumed rather than when the pattern was fitted. A significant Chi-square test indicates that the assumption of a simplex pattern was rejected. Subsequently, an epsilon can be estimated only from a covariance matrix without fitting the serial correlation pattern. The proportions reported above are the combined proportion of such rejections and the cases where $\hat{\epsilon} \geq \hat{\epsilon}_\rho$.

Table 3. Mean Sample Correction Factors and the Proportion of Sample Covariance Matrices Producing Larger Correction Factors when the Serial Correlation Pattern is not Assumed: $t=7$

	ρ					<u>Lowerbound</u>
	0.000	0.250	0.500	0.750	0.900	correction factor
<u>t = 7 at n = 12</u>						
$\hat{\epsilon}$	0.636	0.612	0.541	0.450	0.393	0.167
$\hat{\epsilon}_\rho$	<u>0.980</u>	<u>0.914</u>	<u>0.753</u>	<u>0.558</u>	<u>0.445</u>	<u>0.381</u>
$\hat{\epsilon} \geq \hat{\epsilon}_\rho$	0.000	0.000	0.009	0.108	0.234	
χ^2 sig at .05	0.461	0.470	0.499	0.555	0.604	
χ^2 sig at .10	0.588	0.595	0.612	0.660	0.706	
χ^2 sig at .20	0.732	0.727	0.755	0.781	0.795	
<u>t = 7 at n = 24</u>						
$\hat{\epsilon}$	0.783	0.738	0.631	0.497	0.431	0.167
$\hat{\epsilon}_\rho$	<u>0.991</u>	<u>0.920</u>	<u>0.748</u>	<u>0.553</u>	<u>0.445</u>	<u>0.381</u>
$\hat{\epsilon} \geq \hat{\epsilon}_\rho$	0.000	0.001	0.038	0.216	0.336	
χ^2 sig at .05	0.164	0.181	0.203	0.359	0.446	
χ^2 sig at .10	0.268	0.280	0.309	0.430	0.520	
χ^2 sig at .20	0.401	0.438	0.457	0.546	0.621	

Note: The results are based on 2000 replications. Each of the population covariance matrix had a serial correlation pattern. The proportion in the row $\hat{\epsilon} \geq \hat{\epsilon}_\rho$ indicates the proportion of replications that showed larger correction factors when the simplex pattern was not assumed rather than when the pattern was fitted. A significant Chi-square test indicates that the assumption of a simplex pattern was rejected. Subsequently, an epsilon can be estimated only from a covariance matrix without fitting the serial correlation pattern. The proportions reported above are the combined proportion of such rejections and the cases where $\hat{\epsilon} \geq \hat{\epsilon}_\rho$.

Table 4. Mean Sample Correction Factors and the Proportion of Sample Covariance Matrices Producing Larger Correction Factors when the Serial Correlation Pattern is not Assumed: $t=3$

	ρ					Lowerbound correction factor
	0.000	0.250	0.500	0.750	0.900	
<u>$t = 3$ at $n = 6$</u>						
$\hat{\epsilon}$	0.758	0.752	0.741	0.712	0.687	0.500
$\hat{\epsilon}_\rho$	0.961	0.959	0.929	0.880	0.840	0.800
$\hat{\epsilon} \geq \hat{\epsilon}_\rho$	0.058	0.060	0.105	0.136	0.145	
χ^2 sig at .05	0.424	0.411	0.465	0.533	0.666	
χ^2 sig at .10	0.526	0.535	0.570	0.640	0.749	
χ^2 sig at .20	0.661	0.663	0.692	0.752	0.838	
<u>$t = 3$ at $n = 12$</u>						
$\hat{\epsilon}$	0.834	0.829	0.814	0.767	0.719	0.500
$\hat{\epsilon}_\rho$	0.981	0.969	0.931	0.874	0.836	0.800
$\hat{\epsilon} \geq \hat{\epsilon}_\rho$	0.043	0.086	0.169	0.204	0.154	
χ^2 sig at .05	0.387	0.427	0.518	0.661	0.812	
χ^2 sig at .10	0.514	0.551	0.638	0.753	0.883	
χ^2 sig at .20	0.658	0.689	0.758	0.857	0.940	
<u>$t = 3$ at $n = 24$</u>						
$\hat{\epsilon}$	0.879	0.873	0.847	0.795	0.722	0.500
$\hat{\epsilon}_\rho$	0.991	0.974	0.932	0.872	0.833	0.800
$\hat{\epsilon} \geq \hat{\epsilon}_\rho$	0.028	0.110	0.195	0.207	0.092	
χ^2 sig at .05	0.575	0.637	0.734	0.854	0.977	
χ^2 sig at .10	0.704	0.756	0.823	0.914	0.993	
χ^2 sig at .20	0.826	0.855	0.901	0.964	0.997	

Note: The results are based on 2000 replications. Each of the population covariance matrix had a nonstationary time series structure where the variances increased from 1 to t . The proportion in the row $\hat{\epsilon} \geq \hat{\epsilon}_\rho$ indicates the proportion of replications that showed larger correction factors when the simplex pattern was not assumed rather than when the pattern was fitted. A significant Chi-square test indicates that the assumption of a simplex pattern was rejected. Subsequently, an epsilon can be estimated only from a covariance matrix without fitting the serial correlation pattern. The proportions reported above are the combined proportion of such rejections and the cases where $\hat{\epsilon} \geq \hat{\epsilon}_\rho$.

Table 5. Mean Sample Correction Factors and the Proportion of Sample Covariance Matrices Producing Larger Correction Factors when the Serial Correlation Pattern is not Assumed: $t=5$

	ρ					<u>Lowerbound</u>
	0.000	0.250	0.500	0.750	0.900	correction factor
<u>t = 5 at n = 6</u>						
$\hat{\epsilon}$	0.536	0.531	0.510	0.468	0.423	0.250
$\hat{\epsilon}_\rho$	<u>0.950</u>	<u>0.919</u>	<u>0.827</u>	<u>0.691</u>	<u>0.601</u>	<u>0.526</u>
$\hat{\epsilon} \geq \hat{\epsilon}_\rho$	0.000	0.001	0.003	0.020	0.046	
χ^2 sig at .05	0.847	0.834	0.847	0.869	0.920	
χ^2 sig at .10	0.902	0.894	0.902	0.923	0.951	
χ^2 sig at .20	0.945	0.942	0.946	0.956	0.972	
<u>t = 5 at n = 12</u>						
$\hat{\epsilon}$	0.667	0.654	0.610	0.541	0.465	0.250
$\hat{\epsilon}_\rho$	<u>0.976</u>	<u>0.932</u>	<u>0.819</u>	<u>0.681</u>	<u>0.593</u>	<u>0.526</u>
$\hat{\epsilon} \geq \hat{\epsilon}_\rho$	0.000	0.004	0.019	0.070	0.074	
χ^2 sig at .05	0.610	0.613	0.641	0.740	0.880	
χ^2 sig at .10	0.733	0.736	0.756	0.832	0.930	
χ^2 sig at .20	0.848	0.851	0.866	0.918	0.973	
<u>t = 5 at n = 24</u>						
$\hat{\epsilon}$	0.752	0.731	0.666	0.574	0.479	0.250
$\hat{\epsilon}_\rho$	<u>0.987</u>	<u>0.939</u>	<u>0.817</u>	<u>0.675</u>	<u>0.588</u>	<u>0.526</u>
$\hat{\epsilon} \geq \hat{\epsilon}_\rho$	0.000	0.003	0.035	0.110	0.071	
χ^2 sig at .05	0.828	0.838	0.885	0.944	0.988	
χ^2 sig at .10	0.905	0.913	0.937	0.970	0.995	
χ^2 sig at .20	0.958	0.962	0.976	0.991	0.999	

Note: The results are based on 2000 replications. Each of the population covariance matrix had a nonstationary time series structure where the variances increased from 1 to t . The proportion in the row $\hat{\epsilon} \geq \hat{\epsilon}_\rho$ indicates the proportion of replications that showed larger correction factors when the simplex pattern was not assumed rather than when the pattern was fitted. A significant Chi-square test indicates that the assumption of a simplex pattern was rejected. Subsequently, an epsilon can be estimated only from a covariance matrix without fitting the serial correlation pattern. The proportions reported above are the combined proportion of such rejections and the cases where $\hat{\epsilon} \geq \hat{\epsilon}_\rho$.

Table 6. Mean Sample Correction Factors and the Proportion of Sample Covariance Matrices Producing Larger Correction Factors when the Serial Correlation Pattern is not Assumed: $t=7$

	ρ					Lowerbound correction factor
	0.000	0.250	0.500	0.750	0.900	
<u>$t = 7$ at $n = 12$</u>						
$\hat{\epsilon}$	0.570	0.553	0.503	0.422	0.351	0.167
$\hat{\epsilon}_\rho$	<u>0.977</u>	<u>0.915</u>	<u>0.758</u>	<u>0.564</u>	<u>0.446</u>	<u>0.381</u>
$\hat{\epsilon} \geq \hat{\epsilon}_\rho$	0.000	0.000	0.003	0.038	0.067	
χ^2 sig at .05	0.867	0.879	0.880	0.905	0.951	
χ^2 sig at .10	0.922	0.933	0.934	0.949	0.980	
χ^2 sig at .20	0.966	0.972	0.970	0.977	0.993	
<u>$t = 7$ at $n = 24$</u>						
$\hat{\epsilon}$	0.677	0.654	0.576	0.459	0.368	0.167
$\hat{\epsilon}_\rho$	<u>0.989</u>	<u>0.920</u>	<u>0.753</u>	<u>0.559</u>	<u>0.452</u>	<u>0.381</u>
$\hat{\epsilon} \geq \hat{\epsilon}_\rho$	0.000	0.000	0.005	0.059	0.074	
χ^2 sig at .05	0.956	0.961	0.965	0.981	0.999	
χ^2 sig at .10	0.983	0.985	0.984	0.991	1.000	
χ^2 sig at .20	0.995	0.993	0.995	0.998	1.000	

Note: The results are based on 2000 replications. Each of the population covariance matrix had a nonstationary time series structure where the variances increased from 1 to t . The proportion in the row $\hat{\epsilon} \geq \hat{\epsilon}_\rho$ indicates the proportion of replications that showed larger correction factors when the simplex pattern was not assumed rather than when the pattern was fitted. A significant Chi-square test indicates that the assumption of a simplex pattern was rejected. Subsequently, an epsilon can be estimated only from a covariance matrix without fitting the serial correlation pattern. The proportions reported above are the combined proportion of such rejections and the cases where $\hat{\epsilon} \geq \hat{\epsilon}_\rho$.